Confounder selection via iterative graph expansion

F. Richard Guo

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Statistical Laboratory, University of Cambridge

Qingyuan Zhao

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Introduction

Confounder selection

In observational studies, the single most widely used to strategy to control for confounding is through confounder / covariate **adjustment**.

- Suppose X is a (point) treatment and Y is an outcome.

☞ Adjust for **a set of covariates S** such that

$$Y(x) \perp\!\!\!\perp X \mid S, \quad \text{for every treatment level } x.$$

▶ conditional exchangeability

☞ Then, under positivity,

$$p(Y(x) \mid X = x, S) = p(Y \mid X = x, S).$$

☞ **Confounder selection**: How to select S?

Objectives

1. **Primary**: Finding a set of **observed** covariates S that satisfies conditional exchangeability $Y(x) \perp\!\!\!\perp X \mid S$.
   - Such a set is called a **sufficient adjustment set**.
2. **Secondary**: When there are more than one sufficient adjustment sets, choose one among them to optimize some criterion, such as
   - efficiency
   - cardinality
   - cost
   - ...

☞ We only focus on the **primary objective** in this talk.

- See also Guo, Lundborg, and Zhao (2022) for a recent survey.

A solved problem?

☞ Suppose we want to find a sufficient adjustment set in the causal model[1] represented by a DAG (▶ or ADMG) $\mathcal{G}$ over vertex set V.

Back-door Criterion (Pearl, 1993) $S \subseteq \mathrm{V} \setminus \{X, Y\}$ is a sufficient adjustment set if

1. S contains no descendant of X,
2. there is no '$X \leftarrow \dots$' path between X and Y that is m-connected given S.

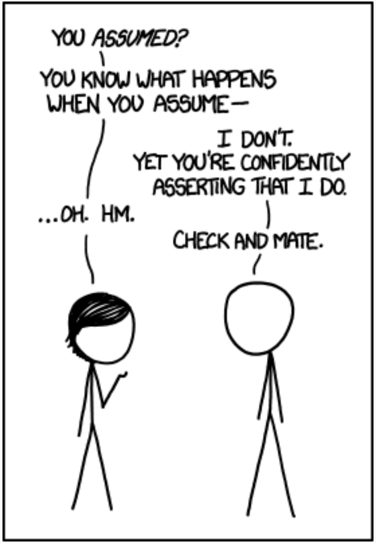
Completeness (Shpitser, VanderWeele, and Robins, 2010) If S is a sufficient adjustment set under the causal model $\mathcal{G}$, then $S \setminus \mathrm{de}(X)$ satisfies the backdoor criterion.

- WLOG, we only consider adjustment sets that contain **no descendants** of X, then

$$S \text{ is a sufficient adjustment set} \iff S \text{ meets the back-door criterion.}$$

[1] Formally, the FFRCISTG/SWIG model, which is a supermodel of NPSEM-IE.

- *"Suppose $\mathcal{G}$ is the causal DAG/ADMG ..."*



https://xkcd.com/1339/

Critiques on the 'known $\mathcal{G}$' stance

1. **Impractical**
   - Do not know the full causal structure/mechanism
   - Even if we know it, can we readily draw it?
     - Where is the boundary?
     - Can you draw the floor plan of your home?
   - Ask a domain expert to draw it
     - Tools and protocols (Shrier and Platt, 2008; Haber et al., 2022) are developed, but still challenging.
2. **Unnecessary**: Back-door criterion only concerns **partial knowledge** about $\mathcal{G}$.

▶ **Two questions**

(i) Can we represent this partial knowledge in a modular format? (▶ representation)

(ii) How do we elicit this partial knowledge? (▶ procedure design)

Earlier proposals

Disjunctive criterion (VanderWeele and Shpitser, 2011):

$$S := (\mathrm{an}_{\mathcal{G}}(X) \cup \mathrm{an}_{\mathcal{G}}(Y)) \cap \{\text{observed pre-treatment covariates}\}.$$

- See also VanderWeele (2019) for its variation.

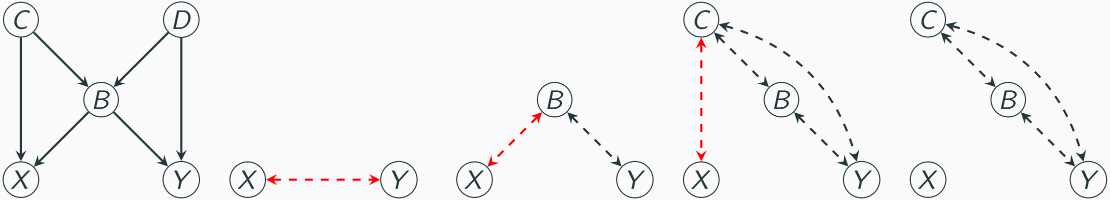
☞ Such S is a **sufficient adjustment set** whenever {observed pre-treatment covariates} contains **any** sufficient adjustment set.

☞ Most useful when

(1) data is already collected,

(2) structural knowledge is scarce.

Our approach in a nutshell



(i) Can we represent this partial knowledge in a modular format? (▶ representation)

☞ Graph with bidirected edges: '← - →' denotes a (potential) **confounding arc**

(ii) How do we elicit this partial knowledge? (▶ procedure design)

☞ Select a confounding arc and introduce new variables (▶ **primary adjustment set**) to control it. (▶ knowledge about **common causes** and **mediators**)

☞ Remove the old arc and adds new arcs.

☞ Iterate until X and Y are disconnected — a sufficient adjustment set is found!

(▶ inverse of latent projection)

Demo time

https://ricguo.shinyapps.io/InteractiveConfSel/

Example

Features of the procedure

- **Confounding arcs**

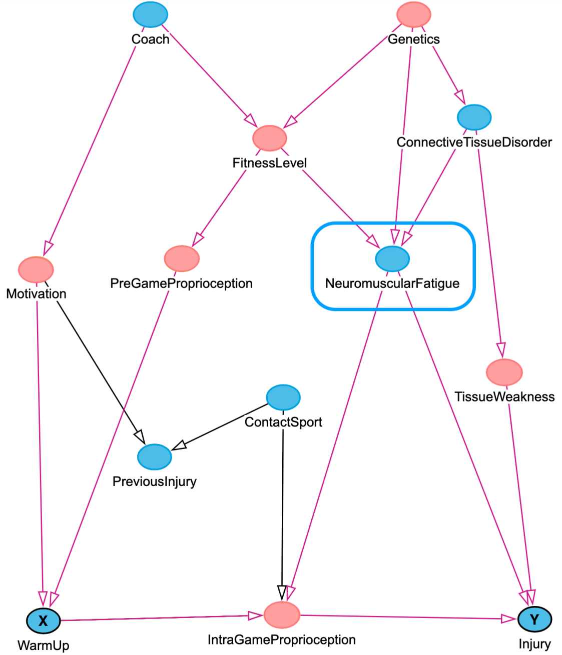
  ⇠⇢: potential confounding arc

  ⟷: uncontrollable confounding arc (▶ no primary adjustment set exists)

- To control ⇠⇢, knowledge is elicited to find its **primary adjustment set**.
  - Does not need pre-specification of the full graph.
  - User answers questions about **common causes** and **mediators**. (▶ Local structures.)
  - **Economical** queries: causal relations between the observed confounders are irrelevant and never asked about!

- User's familiarity with causal graphical models is **not** a prerequisite.

  ▶ Need not be aware of **collider bias** — it is taken care of!


Coach
Genetics
ConnectiveTissueDisorder
FitnessLevel
Motivation
PreGameProprioception
NeuromuscularFatigue
TissueWeakness
ContactSport
PreviousInjury
X
WarmUp
IntraGameProprioception
Y
Injury

Features of the procedure

- **Confounding arcs**

  $\leftarrow\!-\!-\!\rightarrow$: potential confounding arc;

  $\longleftrightarrow$: uncontrollable confounding arc

- To control $\leftarrow\!-\!-\!\rightarrow$, knowledge is elicited to find its **primary adjustment set**.
  - Does not need pre-specification of the full graph.
  - User answers questions about **common causes** and **mediators**. (▶ Local structures.)
  - **Economical** queries: causal relations between the observed confounders are irrelevant and never asked about!

- User's familiarity with causal graphical models is **not** a prerequisite.

  ▶ Need not be aware of **collider bias** — it is taken care of!

- Procedure is terminated when X and Y are

  ▶ disconnected by $\leftarrow\!-\!-\!\rightarrow$ / $\longleftrightarrow$ edges: a sufficient adjustment set is found;

  ▶ connected by $\longleftrightarrow$ edges: no sufficient adjustment set can be found.

Theory

Setting

Let $\mathcal{G}$ be an underlying acyclic directed mixed graph (ADMG) over vertex set $\mathbf{V}$ that defines our causal model.

- An ADMG is a graph with directed ($\rightarrow$) and bidirected edges ($\leftrightarrow$) that has no directed cycle.
  - '$\rightarrow$' represents a (direct) causal effect.
  - '$\leftrightarrow$' represents the existence of a latent common cause, i.e., *endogeneity*.

m-connection/separation (Richardson, 2003) is a direct extension of d-connection/separation:

- A path from A to B is m-connected given C if every non-collider on the path is not in C, and every collider on the path is in C or has a descendant in C.

$$A \perp\!\!\!\perp_m B \mid C \iff \nexists \text{ m-connected path between } A \text{ and } B \text{ given } C$$
$$A \not\perp\!\!\!\perp_m B \mid C \iff \exists \text{ m-connected path between } A \text{ and } B \text{ given } C$$

☞ Graph $\mathcal{G}$ is **unknown**. But we can make **repeated queries** about certain structures in $\mathcal{G}$.

Notation: shapes of paths

Arc '$\sim\!\sim$' = a sequence of adjacent edges with no colliders

- ☞ Directed arc $A \rightsquigarrow B$: $A \to \cdots \to B$
- ☞ **Confounding arc** $A \leftrightsquigarrow B$:

$$A \leftarrow \cdots \to B \text{ or } A \leftarrow \cdots \leftarrow \circ \leftrightarrow B \text{ or } A \leftrightarrow \circ \to \cdots \to B \text{ or } A \leftarrow \cdots \leftrightarrow \cdots \to B$$

Half-arrow = either an endpoint tail or an endpoint head

- ☞ $A \leftharpoonup\!\!\rightsquigarrow B$: $A \rightsquigarrow B$ or $A \leftrightsquigarrow B$
- ☞ $A \leftharpoonup\!\!\sim\!\!\rightharpoondown B$: $A \rightsquigarrow B$ or $A \leftrightsquigarrow B$ or $A \leftarrow\!\!\sim B$

Wildcard $*$ = concatenation of arcs (▶ Imagine '$*$' as zero, one or more colliders)

- ☞ **Confounding path** $A \leftrightsquigarrow * \leftrightsquigarrow B$: $A \leftrightsquigarrow B$ or $A \leftrightsquigarrow \circ \leftrightsquigarrow B$ or $A \leftrightsquigarrow \circ \leftrightsquigarrow \ldots \leftrightsquigarrow B$
- ☞ $A \leftharpoonup\!\!\rightsquigarrow * \leftarrow\!\!\sim\!\!\rightharpoondown B$: path of any shape between A and B

Notation: refined m-connection

$A \langle \text{shape} \rangle B \mid C \iff \exists$ a path of $\langle$shape$\rangle$ between A and B that is m-connected given C,

$A \langle \text{sh}\not{a}\text{pe} \rangle B \mid C \iff \nexists$ a path of $\langle$shape$\rangle$ between A and B that is m-connected given C.

(▶ When $C = \emptyset$, '$A \langle \text{shape} \rangle B \mid \emptyset$' is not be shortened to '$A \langle \text{shape} \rangle B$'.)

☞ **m-connection and m-separation** (▶ '$\leftsquigarrow\!\!\rightsquigarrow * \leftsquigarrow\!\!\rightsquigarrow$' is a path of any shape)

$$A \leftsquigarrow\!\!\rightsquigarrow * \leftsquigarrow\!\!\rightsquigarrow B \mid C \iff A \not\perp\!\!\!\perp_m B \mid C \quad \text{and} \quad A \leftsquigarrow\!\!\rightsquigarrow \not* \leftsquigarrow\!\!\rightsquigarrow B \mid C \iff A \perp\!\!\!\perp_m B \mid C.$$

Refined m-connection

$$A \left\{ \begin{array}{c} \rightsquigarrow \\ \leftsquigarrow\!\!\rightsquigarrow \\ \leftsquigarrow\!\!\rightsquigarrow * \leftsquigarrow\!\!\rightsquigarrow \end{array} \right\} B \mid C$$

'$\leftsquigarrow\!\!\rightsquigarrow$': **confounding arc** (▶ '$\leftarrow\text{-}\text{-}\rightarrow$ / $\longleftrightarrow$' in our procedure)

'$\leftsquigarrow\!\!\rightsquigarrow * \leftsquigarrow\!\!\rightsquigarrow$': **confounding path**

Back-door criterion, reformulated

Back-door criterion, reformulated For any $S \subset \mathrm{V} \setminus \mathrm{de}(X)$,

$$S \text{ satisfies the back-door criterion} \quad \Longleftrightarrow \quad X \leftrightsquigarrow \not* \leftrightsquigarrow Y \mid S.$$

(▶ Suppose $X \rightsquigarrow Y$ in $\mathcal{G}$.)

☞ This is complicated by **collider bias**

$$A \leftrightsquigarrow \not* \leftrightsquigarrow B \mid C \quad \not\Longrightarrow \quad A \leftrightsquigarrow \not* \leftrightsquigarrow B \mid C', \quad C \subset C'.$$

☞ However, **confounding arc** is free of such issue

$$A \not\leftrightsquigarrow B \mid C \quad \Longrightarrow \quad A \not\leftrightsquigarrow B \mid C', \quad C \subset C'.$$

As our notation suggests, any m-connected confounding path '$\leftrightsquigarrow * \leftrightsquigarrow$' is one or more m-connected confounding arcs '$\leftrightsquigarrow$'.

☞ **Main idea**: block all confounding paths by **blocking confounding arcs**, one at at a time.

Primary adjustment set

Set C is a **primary adjustment set** for A, B **relative to** S if $C \cap (\mathrm{de}(A) \cup \mathrm{de}(B)) = \emptyset$ and

$$A \not\leftrightsquigarrow B \mid S, C.$$

When $S = \emptyset$, C is simply called a primary adjustment set for A, B.

(▶ S is a posited adjustment set)

☞ To block $X \leftrightsquigarrow * \leftrightsquigarrow Y \mid S$, each time, we find a primary adjustment set for a constituent confounding arc and add it to S.

▶ $\{X, Y\} \cup S$ is growing — how do we keep track of confounding relations?

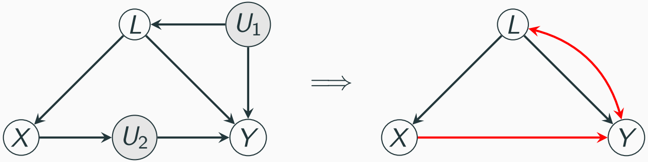
Questions?

Latent projection

☞ Causal model represented by ADMG is closed under **latent projection**.

For $\mathrm{V} = \tilde{\mathrm{V}} \cup \mathrm{U}$ and $\tilde{\mathrm{V}} \cap \mathrm{U} = \emptyset$, let $\mathcal{G}(\tilde{\mathrm{V}})$ be the latent projection of $\mathcal{G}$ onto the margin $\tilde{\mathrm{V}}$.

$$A \left\{ \begin{matrix} \overset{\text{via } \mathrm{U}}{\rightsquigarrow} \\ \overset{\text{via } \mathrm{U}}{\leftsquigarrow} \\ \overset{\text{via } \mathrm{U}}{\leftrightsquigarrow} \end{matrix} \right\} B \; [\mathcal{G}] \quad \Longleftrightarrow \quad A \left\{ \begin{matrix} \rightarrow \\ \leftarrow \\ \leftrightarrow \end{matrix} \right\} B \; [\mathcal{G}(\tilde{\mathrm{V}})].$$



Preservation of refined m-connection

m-connection is preserved by latent projection

$$A \leftrightsquigarrow * \leftrightsquigarrow B \mid C\ [\mathcal{G}(\tilde{\mathrm{V}})] \quad \Longleftrightarrow \quad A \leftrightsquigarrow * \leftrightsquigarrow B \mid C\ [\mathcal{G}], \quad \{A, B\} \cup C \subseteq \tilde{\mathrm{V}} \subseteq \mathrm{V}\,.$$

Theorem For any ADMG $\mathcal{G}$ over vertex set V and $\{A, B\} \cup C \subseteq \tilde{\mathrm{V}} \supseteq \mathrm{V}$,

$$A \left\{ \begin{array}{c} \rightsquigarrow \\ \leftrightsquigarrow \\ \leftrightsquigarrow * \leftrightsquigarrow \end{array} \right\} B \mid C\ [\mathcal{G}] \quad \Longleftrightarrow \quad A \left\{ \begin{array}{c} \rightsquigarrow \\ \leftrightsquigarrow \\ \leftrightsquigarrow * \leftrightsquigarrow \end{array} \right\} B \mid C\ [\mathcal{G}(\tilde{V})].$$

(▶ Relations induced by $\not\leftrightsquigarrow$ and $\leftrightsquigarrow \not* \leftrightsquigarrow$ are weaker than semi-graphoids.)

Corollary For $A, B \notin C$,

$$A \left\{ \begin{array}{c} \rightsquigarrow \\ \leftrightsquigarrow \\ \leftrightsquigarrow * \leftrightsquigarrow \end{array} \right\} B \mid C\ [\mathcal{G}] \quad \Longleftrightarrow \quad A \left\{ \begin{array}{c} \rightarrow \\ \leftrightarrow \\ \leftrightarrow * \leftrightarrow \end{array} \right\} B\ [\mathcal{G}(\{A, B\} \cup C)].$$

Representation and procedure

District criterion

$$\underbrace{S \text{ is a sufficient adjustment set}}_{\text{block all back-door paths in } \mathcal{G}} \iff \underbrace{X \leftrightarrow \not{*} \leftrightarrow Y \mid S\, [\mathcal{G}(\{X, Y\} \cup S)]}_{\text{connectivity by confounding arcs}}.$$

☞ This leads to an explicit **representation** and a **procedure**.

- For a posited S, for every $u, v \in \bar{S} := \{X, Y\} \cup S$, maintain **edges** as **m-connected confounding arcs** (▶ directed edges are irrelevant):

$$u \longleftrightarrow v \quad \iff \quad u \leftrightsquigarrow v \mid \bar{S} \setminus \{u, v\}.$$

- An edge is initially drawn as a potential confounding arc '$\leftarrow - \rightarrow$'.
  If there exists a **primary adjustment set** to control it, the edge is removed;
  Otherwise, the edge becomes an uncontrollable confounding arc '$\longleftrightarrow$'.
- Primary adjustment set can be constructed from local knowledge about **common causes** and **mediators**.

$\mathcal{R} = \{\}$ ▷ Set of sufficient adjustment sets
$\mathcal{Q} = \textsc{PriorityQueue}((\emptyset, \emptyset, \emptyset))$ ▷ Initial graph has a possible edge $X \leftarrow - \rightarrow Y$
while $\mathcal{Q} \neq \emptyset$ **do**
  $(S, \mathcal{B}_y, \mathcal{B}_n) = \textsc{Pop}(\mathcal{Q})$ ▷ Pop a graph with smallest min-cut(X, Y)
  $\bar{S} = S \cup \{X, Y\}$
  if $X \leftrightarrow * \leftrightarrow Y$ by edges in $\mathcal{B}_y$ **then** ▷ Fails the district criterion
    continue
  else if $X \leftrightarrow \not* \leftrightarrow Y$ by edges in $(\bar{S} \times \bar{S}) \setminus \mathcal{B}_n$ **then** ▷ Satisfies the district criterion
    $\mathcal{R} = \mathcal{R} \cup \{S\}$
    continue
  end if
  $(A, B) = \pi = \textsc{SelectEdge}(X,\ Y,\ S,\ \mathcal{B}_y,\ \mathcal{B}_n)$ ▷ π is selected from $(\bar{S} \times \bar{S}) \setminus (\mathcal{B}_y \cup \mathcal{B}_n)$
  $\mathcal{L} = \textsc{FindPrimary}((A, B); S \setminus \{A, B\})$
  if $\emptyset \in \mathcal{L}$ **then**
    $\textsc{Push}(\mathcal{Q},\ (S,\ \mathcal{B}_y,\ \mathcal{B}_n \cup \{\pi\}))$ ▷ π need not be expanded
  else
    for $C \in \mathcal{L}$ **do**
      $\textsc{Push}(\mathcal{Q},\ (S \cup C,\ \mathcal{B}_y,\ \mathcal{B}_n \cup \{\pi\}))$ ▷ Expand π by each primary adjustment set
    end for
    $\textsc{Push}(\mathcal{Q},\ (S,\ \mathcal{B}_y \cup \{\pi\},\ \mathcal{B}_n))$ ▷ Not to expand π
  end if
end while
return $\mathcal{R}$

Soundness and completeness

☞ If running the procedure **exhaustively** (▶ with a stack or a queue) with the input answered by an **oracle** subroutine $\textsc{FindPrimary}((A, B); S')$, the following can be shown.

Theorem Consider any two vertices X, Y in an ADMG $\mathcal{G}$ such that $X \rightsquigarrow Y$.

1. **Soundness (primary ⇒ sufficiency)**: Suppose every $C \in \textsc{FindPrimary}((A, B); S')$ is a primary adjustment set for A, B given S' in $\mathcal{G}$. Then every element in the output of $\textsc{ConfounderSelect}(X, Y)$ is a sufficient adjustment set for (X, Y).
2. **Completeness (all minimal primary ⇒ all minimal sufficiency)**: Suppose further that $\textsc{FindPrimary}((A, B); S')$ contains all minimal primary adjustment sets for (A, B) given S' in $\mathcal{G}$. Then the output of $\textsc{ConfounderSelect}(X, Y)$ contains all minimal sufficient adjustment sets for (X, Y).

▶ 'Minimal': no proper subset is also a primary/sufficient adjustment set.

Summary

- Confounding **path** '$\leftarrow\!\!\!\rightsquigarrow * \leftarrow\!\!\!\rightsquigarrow$' provides a structural definition of **confounding**.
  - ▶ Confounding arcs '$\leftarrow\!\!\!\rightsquigarrow$' are the building blocks.
  - ▶ Related: definition of a confounder (VanderWeele and Shpitser, 2013)
- **Economical queries**: causal relations between variables in S are never elicited.
- The user answers questions about **common causes** and **mediators**. Need not be aware of **collider bias**.
  - ▶ Compared to undirected graphs, colliders can be an extra complication to beginners in graphical models.
  - ▶ This was an issue of debate in the literature (Shrier, 2008; Rubin, 2009; Pearl, 2009; Sjölander, 2009).
- Moving away from the "known $\mathcal{G}$" stance: typically, **identification** only relies on certain **partial knowledge** the underlying causal graph.
  - ▶ How to **represent** such knowledge? ▶ How to elicit it?
  - ▶ More development is needed on how to use causal graphs to **design** a study.

Give it a try!

https://ricguo.shinyapps.io/InteractiveConfSel/

arxiv: 2309.06053

References

- Guo, F. Richard, Anton Rask Lundborg, and Qingyuan Zhao (2022). **"Confounder Selection: Objectives and Approaches".** In: *arXiv preprint arXiv:2208.13871*.
- Pearl, Judea (1993). **"Comment: graphical models, causality and intervention".** In: *Statistical Science* 8.3, pp. 266–269.
- Shpitser, Ilya, Tyler J. VanderWeele, and James M. Robins (2010). **"On the validity of covariate adjustment for estimating causal effects".** In: *Proceedings of the Twenty-Sixth Conference on Uncertainty in Artificial Intelligence*. UAI'10. Arlington, Virginia, USA: AUAI Press, pp. 527–536.

- Shrier, Ian and Robert W Platt (2008). **"Reducing bias through directed acyclic graphs".** In: *BMC Medical Research Methodology* 8, pp. 1–15.
- Haber, Noah A et al. (2022). **"DAG With Omitted Objects Displayed (DAGWOOD): a framework for revealing causal assumptions in DAGs".** In: *Annals of Epidemiology* 68, pp. 64–71.
- VanderWeele, Tyler J. and Ilya Shpitser (2011). **"A new criterion for confounder selection".** In: *Biometrics* 67.4, pp. 1406–1413.
- VanderWeele, Tyler J. (2019). **"Principles of confounder selection".** In: *European Journal of Epidemiology* 34.3, pp. 211–219.
- Richardson, Thomas S. (2003). **"Markov Properties for Acyclic Directed Mixed Graphs".** In: *Scandinavian Journal of Statistics* 30.1, pp. 145–157.

- VanderWeele, Tyler J. and Ilya Shpitser (2013). **"On the Definition of a Confounder".** In: *The Annals of Statistics* 41.1, pp. 196–220.
- Shrier, Ian (2008). **"Letter to the Editor".** In: *Statistics in Medicine* 27.14, pp. 2740–2741.
- Rubin, Donald B. (2009). **"Should observational studies be designed to allow lack of balance in covariate distributions across treatment groups?"** In: *Statistics in Medicine* 28.9, pp. 1420–1423.
- Pearl, Judea (2009). **"Remarks on the method of propensity score".** In: *Statistics in Medicine* 28.9, pp. 1415–1416.
- Sjölander, Arvid (2009). **"Propensity scores and M-structures".** In: *Statistics in Medicine* 28.9, pp. 1416–1420.

References iv

Sadeghi, Kayvan and Steffen Lauritzen (2014). **"Markov properties for mixed graphs".** In: *Bernoulli* 20.2, pp. 676–696.

Graphoid-like properties

- Let $\mathcal{J}$ be a collection of tuples $\langle A, B \mid C\rangle$ for A, B, C that are disjoint subsets of a ground set V. For disjoint $A, B, C, D \subset \mathrm{V}$, consider the following properties:

  (i) triviality: $\langle A, \emptyset \mid C\rangle$ for every disjoint $A, C \subset \mathrm{V}$;
  (ii) symmetry: $\langle A, B \mid C\rangle \implies \langle B, A \mid C\rangle$;
  (iii) decomposition: $\langle A, B \cup C \mid D\rangle \implies \langle A, B \mid D\rangle$ and $\langle A, C \mid D\rangle$;
  (iv) weak union: $\langle A, B \cup C \mid D\rangle \implies \langle A, B \mid C \cup D\rangle$;
  (v) contraction: $\langle A, C \mid D\rangle$ and $\langle A, B \mid C \cup D\rangle \implies \langle A, B \cup C \mid D\rangle$;
  (vi) intersection: $\langle A, B \mid C \cup D\rangle$ and $\langle A, C \mid B \cup D\rangle \implies \langle A, B \cup C \mid D\rangle$;
  (vii) composition: $\langle A, B \mid D\rangle$ and $\langle A, C \mid D\rangle \implies \langle A, B \cup C \mid D\rangle$.

  We say $\mathcal{J}$ is a *semi-graphoid* over V, if it satisfies (i)–(v); further, we say $\mathcal{J}$ is a *graphoid* over V, if it satisfies (i)–(vi), and finally, a *compositional graphoid* over V, if it satisfies (i)–(vii).

- m-separation $\mathcal{J}_{\leftrightsquigarrow \not\perp \leftrightsquigarrow}$ is a compositional graphoid (Sadeghi and Lauritzen, 2014).

☞ Relations

$$\mathcal{J}_{\not\leftrightsquigarrow} := \{\langle A, B \mid C\rangle : A \not\leftrightsquigarrow B \mid C \; [\mathcal{G}]\} \text{ and } \mathcal{J}_{\leftrightsquigarrow \not\perp \leftrightsquigarrow} := \{\langle A, B \mid C\rangle : A \leftrightsquigarrow \not\perp \leftrightsquigarrow B \mid C \; [\mathcal{G}]\}$$

only satisfy properties (i)–(iv) and (vii).